Saturation of low-energy antiproton annihilation on nuclei

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Abstract

Recent measurements of very low-energy ($p_L < 100$ MeV/c) $\bar{p}$ annihilation on light nuclei reveal apparent suppression of annihilation upon increasing the atomic charge Z and mass number A. Using $\bar{p}$-nucleus optical potentials $V_{\rm opt}$, fitted to $\bar{p}$-atom energy-shifts and -widths, we resolve this suppression as due to the strong effective repulsion produced by the very absorptive $V_{\rm opt}$. The low-energy $\bar{p}$-nucleus wavefunction is kept substantially outside the nuclear surface and the resulting reaction cross section saturates as function of the strength of Im $V_{\rm opt}$. This feature, for $E > 0$, parallels the recent prediction, for $E < 0$, that the level widths of $\bar{p}$ atoms saturate and, hence, that $\bar{p}$ deeply bound atomic states are relatively narrow. Low-energy annihilation cross sections of $\bar{p}$ across the periodic table are calculated, and their dependence on Z and A is classified and discussed.

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I. INTRODUCTION

Experimental results for antiproton annihilation cross sections at very low energies ($p_L <$ 100 MeV/c), below the $\bar{p}p \to \bar{n}n$ charge-exchange threshold, have recently been reported for light nuclei [1–3]. At these energies the total $\bar{p}$ reaction cross section consists only of $\bar{p}$ annihilation. As shown in Table 1 of Ref. [3], whereas at relatively higher energies ($p_L \approx 200 - 600$ MeV/c) the ratios of $\bar{p}$ annihilation cross sections on different nuclei exhibit the well-known $A^{2/3}$ strong-absorption dependence, these ratios at very low energies defy any simple, obvious regularity. It has been demonstrated that the *'expected'* $ZA^{1/3}$ dependence on the atomic charge Z and mass number A is badly violated [3,4]. Antiproton annihilation cross sections at very low energies simply do not rise with A as fast as is anticipated. For example, over the whole mass range studied so far (H, D, ^{4}He, Ne), the Ne/H ratio of total annihilation cross sections is at least 6 times smaller than expected.

In the present Letter we study low-energy $\bar{p}$ annihilation on nuclei, using the optical model approach. Optical potentials have been very successful in describing strong-interaction effects in hadronic atoms [5], including $\bar{p}$ atoms [6]. It has been noted, for pions, that the total reaction cross sections at low energies are directly related to the atomic-state widths, and that once a suitable optical potential is constructed by reasonably fitting it to the atomic level shifts and widths in the negative energy bound-state domain, these total reaction cross sections are reliably calculable [7,8]. The recent publications [1–3] of experimental results of total cross sections for $\bar{p}$ annihilation on nuclei at very low energies raise the intriguing possibility of connecting these two energy regimes in a systematic way also for antiprotons. However, most of the data on annihilation cross sections for $\bar{p}$ are for very light nuclei, where the concept of a rather universal optical potential that depends on A and Z only through the nuclear densities is questionable. For this reason we use optical potentials, in the present work, mostly for crossing the $E = 0$ borderline within the *same* atomic mass (A) range, from bound states to scattering. This optical model approach is different from the allegedly model-independent scattering length approximation [9] which we have found to be less useful. We defer a discussion of the latter statement to a separate publication.

Among the targets used in the $\bar{p}$ annihilation measurements [1–3], the only 'real' nuclei are ^{4}He and Ne, in the sense that the conventional optical model cannot be applied to the lighter targets of hydrogen or deuterium, or be used at these very low energies to relate data in heavier targets to data on H and D. In the present work we show that the low-energy $\bar{p}$ total annihilation cross sections on ^{4}He [2] and on Ne [3] are reproduced well by reasonable optical potentials fitted separately to the $\bar{p}$ atomic data at the relevant atomic mass range. These potentials are strongly absorptive, which leads to a remarkable saturation of the total reaction cross section with increasing A. Strong absorption has very recently been shown [10,11] to lead also to saturation of the widths of $\bar{p}$ atomic states and to the prediction of relatively narrow deeply bound $\bar{p}$ atomic states. In view of the close analogy between bound-state widths and total reaction cross sections (see Eqs. (1,2) below), it should come as no surprise that the underlying physics is the same. Lastly, we use $\bar{p}$ optical potentials fitted to a comprehensive set of atomic data across the periodic table to predict $\bar{p}$ total annihilation cross sections at very low energies for a wide range of A values, with the exception of very light targets. We discuss the A and Z dependence of these calculated cross sections, showing explicitly how the $ZA^{1/3}$ dependence is radically altered in realistic cases.

II. OPTICAL POTENTIALS

In the present work we aim at connecting $\bar{p}$ atoms, at energies slightly below threshold, with $\bar{p}$ annihilation on nuclei at very low energies above threshold, using an optical potential $V_{\rm opt}$. Assuming for simplicity a Schrödinger-type equation, the width Γ of an atomic level is given *exactly* by:

$$\frac{\Gamma}{2} = -\frac{\int {\rm Im}V_{\rm opt}(r)|\psi(\mathbf{r})|^2 d\mathbf{r}}{\int |\psi(\mathbf{r})|^2 d\mathbf{r}} \quad . \tag{1}$$

Here $\psi(\mathbf{r})$ is the $\bar{p}$ full atomic wavefunction. The corresponding expression for the total reaction cross section at positive energies is

$$\sigma_R = -\frac{2}{\hbar v}\int |\chi(\mathbf{r})|^2 {\rm Im}V_{\rm opt}(r) d\mathbf{r} \quad . \tag{2}$$

Here $\chi(\mathbf{r})$ is the $\bar{p}$ - nucleus elastic scattering wavefunction and v is the c.m. velocity.

There exist several optical model potentials [5,6] parameterized in terms of nuclear densities which are quite successful in reproducing the strong interaction effects in not too light nuclei, e.g. fitting all data for nuclei heavier than carbon. However, most of the good quality data on $\bar{p}$ annihilation at very low energies are for very light nuclei where a straightforward application of the potential deduced from heavier targets is not expected to yield good fits. We have therefore started the present analysis by studying $\bar{p}$ atoms of ^{3,4}He.

The $\bar{p}$ nucleus optical potential used here is given by the '$t\rho$' expression [5]

$$2\mu V_{\rm opt}(r) = -4\pi(1+\frac{\mu}{m})b_0\rho(r) \quad , \tag{3}$$

where m is the nucleon mass, μ is the reduced $\bar{p}$-nucleus mass, b_0 is a complex parameter obtained from fits to the data and $\rho(r)$ is the nuclear density distribution normalized to A. Trying to fit level shift and width data in ^{3,4}He [12] for the $2p$ and $3d$ states simultaneously, we always ended up with the calculated width of the $2p$ state in ^{4}He being too small, thus contributing an unacceptably large value to the total χ^2 of the fits. Handling each isotope separately did not change this situation and we then decided to fit only the data for the $2p$ states for the two isotopes. This is somewhat unfortunate as one expects to be able to use l-independent potentials, as is indeed the case with heavier targets [5,6]. However, since our main concern here was to use atomic potentials at (very low) positive energies where d and higher partial waves contribute very little in light nuclei, we considered this procedure acceptable. Note that we are using the same potential for all the partial waves in the positive energy regime.

Very good fits to the data with reasonable values for the complex parameter b_0 could be obtained only when a 'finite range' was introduced in the form of a Gaussian folding of a $\bar{p}N$ interaction into the nuclear density distributions. This is in contrast to the case of heavier targets where such a procedure was not necessary. Nevertheless, we consider the resulting potential, with b_0=0.49+i3.0 fm and a Gaussian with a range parameter of 1.4 fm folded into the nuclear density distribution (hereafter referred to as potential (a)), as quite reasonable. It fits within the errors the measured strong interaction shifts and widths of the $2p$ state in antiprotonic ^{3}He and ^{4}He.

III. RESULTS AND DISCUSSION

Turning to positive energies, we first calculate the total reaction (annihilation) cross section for 57 MeV/c $\bar{p}$ on ^{4}He, to compare with the average of the measured cross sections, at 47.0 and 70.4 MeV/c [2], quoted as 915±39 mb in Ref. [3]. Using the above potential, the calculated cross section is 901 mb, in excellent agreement with the 'measured' one. For the actual $\bar{p}$ incident momenta we calculate values of 1060 and 771 mb at 47.0 and 70.4 MeV/c, respectively. The experimental values are 979±145 and 827±38 mb, respectively. Reasonably good agreement is therefore established between experiment and predictions made with a potential derived from fits to $\bar{p}$ atoms of ^{3,4}He.

Figure 1 demonstrates the extreme strong-absorption conditions which are relevant to the $\bar{p}$ nucleus interaction at very low energies (and for $\bar{p}$ atoms). It shows calculated reaction cross sections for $\bar{p}$ at 57 MeV/c on ^{4}He and Ne as function of the strength Im b_0 of the imaginary part of the potential (a) described above, with the rightmost edge corresponding to its nominal value Im b_0=3.0 fm. It is seen that as long as the absorptivity (Im b_0) is very weak, less than 1% of its nominal value, σ_R is approximately linear in Im b_0, which according to Eq. (2) means that the $\bar{p}$ wavefunction depends weakly on Im b_0. However, already at below 5% of the nominal value of Im b_0 the reaction cross sections begin to saturate, much the same as for the widths of deeply bound $\bar{p}$ atomic states [10,11]. The mechanism is the same in both cases, namely exclusion of the wavefunction from the nucleus due to the absorption, which reduces dramatically the overlap with the imaginary potential and consequently reduces the integrals in Eqs. (1,2). The onset of saturation is determined approximately by the strength parameter $2\mu(\mathrm{Im}\, V_{\mathrm{opt}})R^2$, where R is the radius of the nucleus. Thus, saturation of σ_R in Ne starts at a smaller absorptivity than its onset in ^{4}He. The Ne/^{4}He ratio of σ_R values changes, due to this effect, from about 15 in the perturbative regime of very weak absorptivity to about 3 in the strong-absorption regime. However, the precise, detailed pattern of the change also depends on the real part of the optical potential which may increase or reduce the exclusion of the $\bar{p}$ wavefunction from the nucleus.

Figure 2 shows calculated $\bar{p}$ reaction cross sections at 57 MeV/c across the periodic table. The dot-dashed line is for the above mentioned potential (a) which is expected to be valid only in the immediate vicinity of He. The dashed line (b) is for the first potential from Table 7 of Ref. [5] which fits $\bar{p}$ atom data over the whole periodic table, starting with carbon. This potential is not expected to fit data for very light nuclei, and indeed it does not fit the ^{4}He annihilation cross section. However, it is noteworthy that for $A > 20$ the two potentials predict almost the same cross sections and certainly display a very similar dependence on A. Also shown in the figure is the recent experimental result [3] for Ne, with very limited accuracy. Furthermore, for $A > 20$ the points along the solid line are the calculated $\bar{n}$ - nucleus total reaction cross sections, obtained from potential (b) by switching off the Coulomb interaction. The solid line is a fit to an $A^{1/3}$ power law which appears to be appropriate to strong absorption of uncharged particles at very low energies. We checked that by reducing the absorptivity by 3 - 4 orders of magnitude, a proportionality of σ_R to A as expected for weak absorption, is indeed observed. Comparing the dashed line for negatively charged particles with the solid line for uncharged particles, it is clear that the σ_R values obtained by including the Coulomb interaction are considerably enhanced with respect to the ones obtained without it. Attempting to fit this enhancement by a Z^α power

law, we find that α varies roughly between 1/3 for Ne to 1/2 for Pb. We are not aware of any universal regularity describing such enhancement across the periodic table, in agreement with the recent numerical findings of Ref. [13].

Finally, we discuss the $ZA^{1/3}$ scaling considered in Refs. [3,4] and in references quoted therein. We note that an attractive Coulomb potential causes focussing of partial-wave trajectories onto the nucleus, an effect which at low energies may be evaluated semiclassically (see Ref. [14] for the applicability of the semiclassical approximation to Coulomb scattering). The maximal orbital angular momentum $l_{\rm max}$, for which the $\bar{p}$-nucleus distance of closest approach is smaller than the nuclear radius R, at sufficiently low energy is given by

$$(l_{\rm max}+1/2)^2 \approx 2\eta kR \quad , \tag{4}$$

where k is the c.m. momentum and

$$\eta = 2Z/k_L a_{\rm B}^{(p)} \tag{5}$$

is the corresponding Coulomb parameter. Here $k_L = p_L/\hbar$, where p_L is the $\bar{p}$ laboratory momentum, and $a_{\rm B}^{(p)}$ is the Bohr radius of the $\bar{p}p$ atom. Since $2\eta >> kR$ at low energies and for high values of Z, the value of $l_{\rm max}$ greatly exceeds the value kR which is appropriate to uncharged particles ($\eta = 0$). Therefore, for strong absorption, where all the lower partial waves are totally absorbed, one obtains

$$\sigma_R \approx \frac{\pi}{k^2}\sum_{l=0}^{l_{\rm max}}(2l+1) \approx \frac{2\eta}{kR}\pi R^2 = 2\pi\eta\frac{R}{k} \quad , \tag{6}$$

which we rewrite as

$$\sigma_R \approx (4\pi r_0/a_{\rm B}^{(p)})(ZA^{1/3}/kk_L) \quad , \tag{7}$$

with r_0 defined by $R = r_0 A^{1/3}$. A similar approach was used by Fäldt and Pilkuhn [15] to calculate Coulomb corrections to $\pi^{\pm}$-nucleus total cross sections near the 3,3 resonance. In the present case of very low energies their correction becomes the dominant term.

In Figure 3 we plot, for potential (b), ratios of calculated total reaction cross sections to the scaling parameter $S = ZA^{1/3}/kk_L$. It is clear that σ_R does not scale with S across the periodic table. Also plotted is the asymptotic value

$$\sigma_R/S \rightarrow 4\pi r_0/a_{\rm B}^{(p)} \approx 0.273 \quad , \tag{8}$$

with $r_0 = 1.25$ fm and $a_{\rm B}^{(p)} = 57.6$ fm. This asymptotic value is approached very slowly.

Before closing we briefly mention other derivations of the $ZA^{1/3}$ dependence of σ_R. The proportionality to Z is usually derived from the well-known Coulomb enhancement Gamow factor which at low energies is well approximated by $2\pi\eta$ [13]. However, at these low energies, the Coulomb wavefunctions are far from being constant over the nuclear volume, so that the applicability of the Gamow factor appears doubtful. The origin of a possible proportionality of σ_R to $A^{1/3}$ is even less clear: for s-wave dominated $\bar{p}$ annihilation, as considered in Refs. [3,4], it is due to the imaginary part of the $\bar{p}$ scattering length which is assumed to closely follow the nuclear radius R. However, for $\bar{p}$ strongly absorptive potentials this imaginary part has invariably been found to be about 1 fm across the periodic table [16] (see also Ref. [9]), considerably less than R. It is inconceivable that a proportionality to R can arise unless several partial waves contribute significantly.

IV. CONCLUSIONS

In conclusion, we have shown that the recently reported annihilation cross sections for $\bar{p}$ on ^{4}He and Ne at 57 MeV/c are reproduced very well by optical potentials which fit $\bar{p}$ atomic data in the respective mass ranges (Fig. 2). For these strongly absorptive potentials, the $ZA^{1/3}$ dependence of the total reaction cross section was derived, but at 57 MeV/c it was found not to be satisfied across the periodic table except for unrealistically high values of A (Fig. 3). The correct dependence is determined by the saturation property of total reaction cross sections at very low energies (Fig. 1). The apparent suppression of $\bar{p}$ total annihilation cross sections measured at $p_L < 100$ MeV/c is a direct consequence of the strong absorption, and in particular its saturation property. Predictions have been given for $\bar{p}$ total annihilation cross sections at very low energies across the periodic table which would complement and enrich the information deduced from the measurements already available for $\bar{p}$ atoms.

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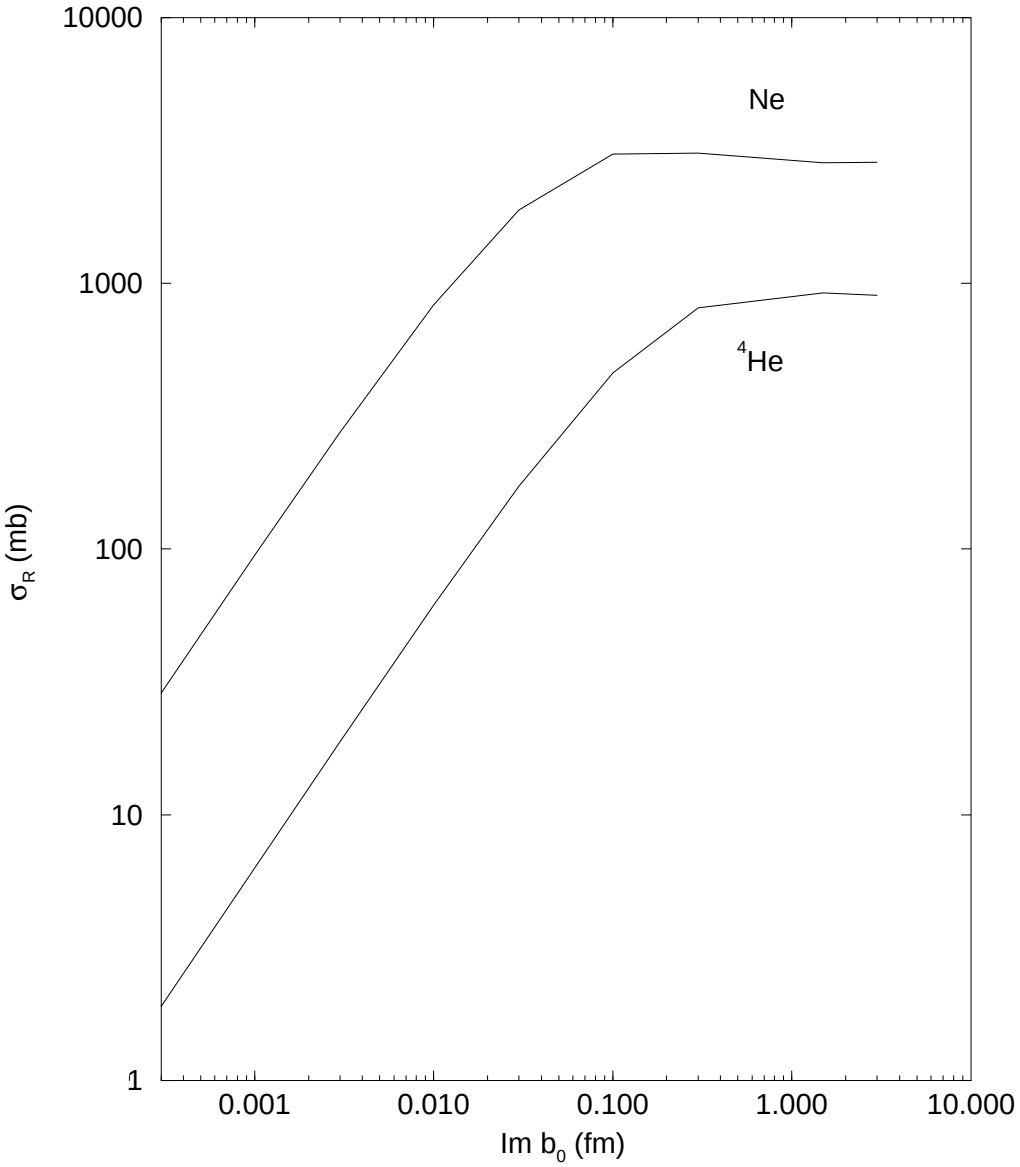

FIG. 1. Calculated total reaction cross sections for 57 MeV/c $\bar{p}$ on ^{4}He and Ne as function of the strength parameter Im b_0 of the optical potential (a).

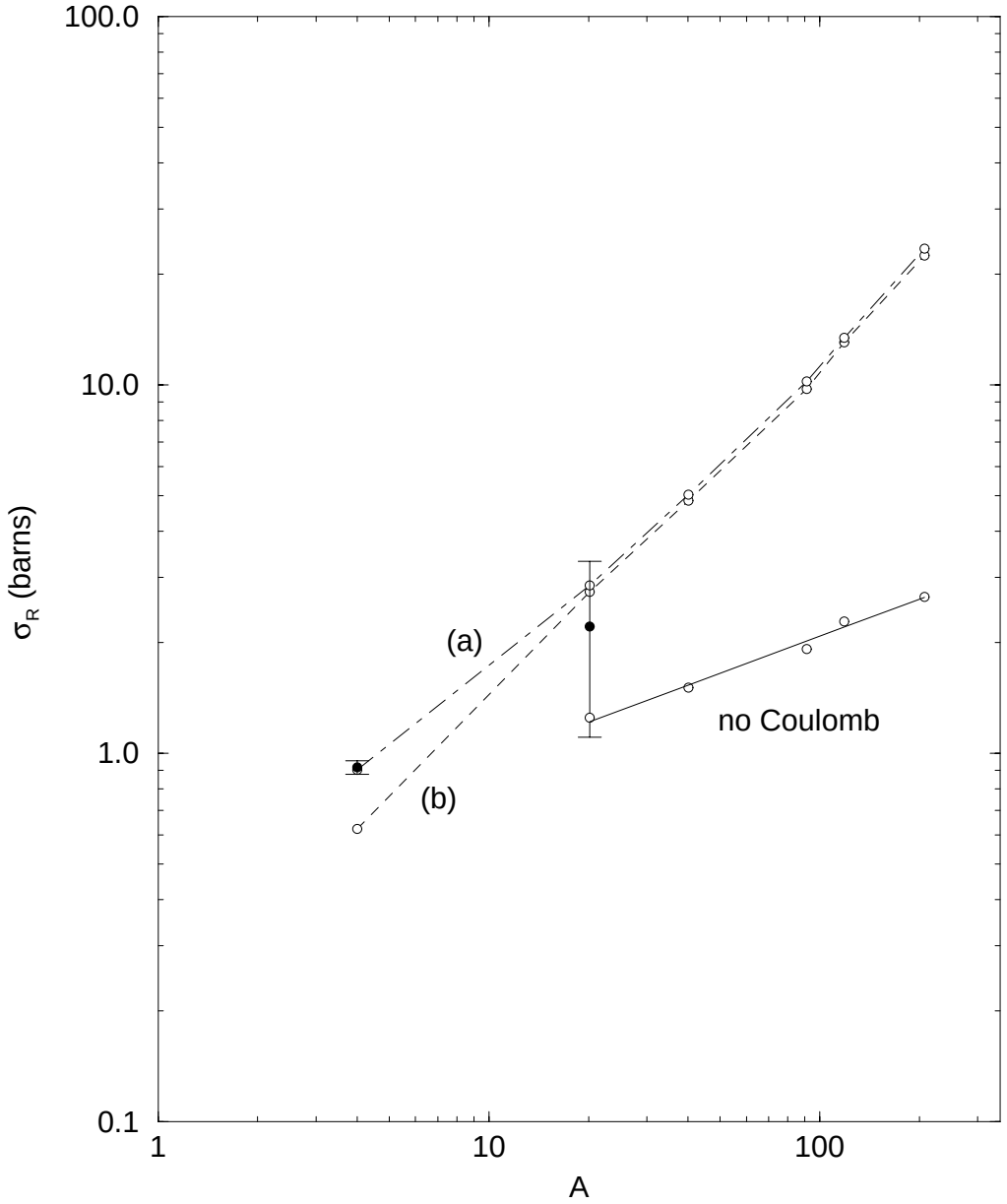

FIG. 2. Calculated $\bar{p}$ total reaction cross sections (open circles) at 57 MeV/c for potentials (a) and (b), and for potential (b) but without the Coulomb interaction. Also shown are the two data points for ^{4}He and Ne.

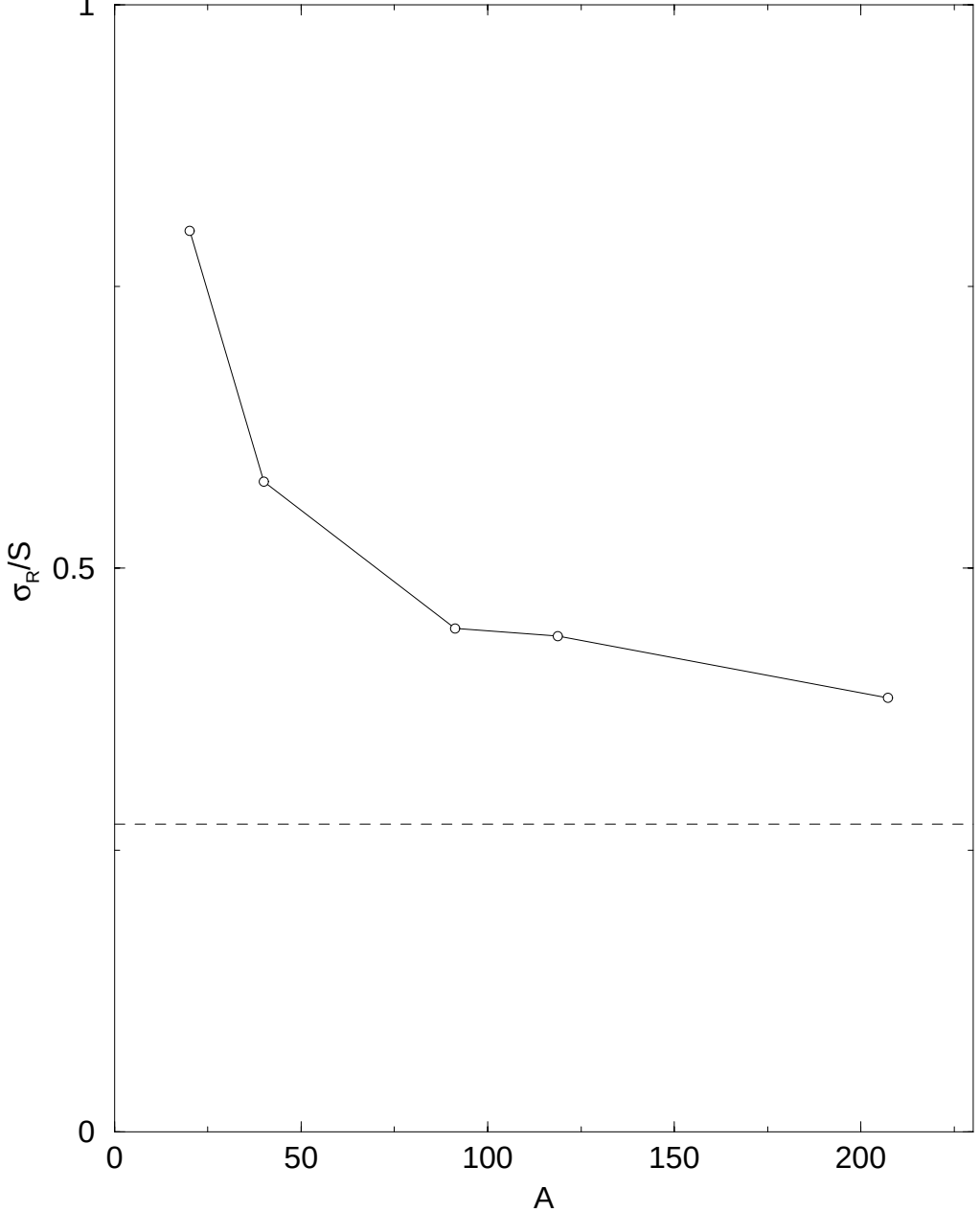

FIG. 3. Ratios of calculated $\bar{p}$ total reaction cross sections at 57 MeV/c, for potential (b), to the scale function S. The dashed straight line provides the asymptotic value expected for σ_R/S, see text.